Forum Teratec 2017
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A FEW CHALLENGES REVISITED AT SCALE

parallel performance v.s. attainable accuracy
robustness to soft errors

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joint work with

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HiePACS - Inria Project
Inria Bordeaux Sud-Ouest

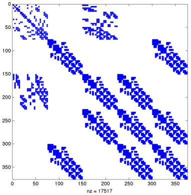
Forewords

HiePACS objectives: Contribute to the design of effective tools for frontier simulations arising from challenging research and industrial multi-scale applications towards extreme computing

- Study and design of novel numerical algorithms for emerging computing platforms
- Analyse their possible weaknesses and possible remedies

Scientific context: numerical linear algebra

Goal: solving $Ax = b$, where A is sparse



Appears in many academic and industrial simulation codes for various engineering applications: accelerator physics, chemical process simulations, earth and environmental sciences, fluid flow, fusion energy, structural analysis, structural biology, ...

- Still promising solution techniques based on Krylov subspace methods (Aleksei Nikolaevich Krylov, 1863-1945)
- Oldest but still effective solver : the conjugate gradient method (CG) [M .R. Hestenes and E. Stiefel, JNRBS, 1952]

1. Improving attainable accuracy

2. Detecting soft error in the Conjugate Gradient method

Outline

1. Improving attainable accuracy
2. Detecting soft error in the Conjugate Gradient method

Original algorithm at a glance

for $i = 0, \ldots$ **do**
    $s_i := Ap_i$
    $\alpha_i := r_i^T u_i / s_i^T p_i$
    $x_{i+1} := x_i + \alpha_i p_i$
    $r_{i+1} := r_i - \alpha_i s_i$
    $u_{i+1} := M^{-1} r_{i+1}$
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- Parallel performance bottleneck: 2 separated synchronizing scalar-products

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- Parallel performance bottleneck: 2 separated synchronizing scalar-products
- Many variants have been designed to overcome this drawback, one of the most recent and promising is pipelined CG
  [P. Ghysels and W. Vanroose, ParCo, 2014]

Pipelined CG - p-CG

for $i = 0, \ldots$ **do**

$\alpha_i := r_i^T u_i \quad \beta_i := w_i^T u_i$

$m_i := M^{-1} w_i \quad v_i := A m_i$

$z_i := v_i + \beta_i z_{i-1} \quad q_i := m_i + \beta_i q_{i-1}$

$s_i := w_i + \beta_i s_{i-1} \quad p_i := u_i + \beta_i p_{i-1}$

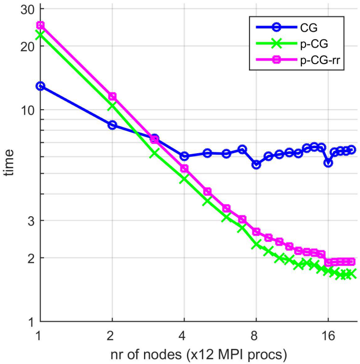
$x_{i+1} := x_i + \alpha_i p_i \quad r_{i+1} := r_i - \alpha_i s_i$

$u_{i+1} := u_i - \alpha_i q_i \quad w_{i+1} := w_i - \alpha_i z_i$

end for

- A single non-synchronizing double scalar-products $\rightarrow$ possible overlap of mat-vec and preconditioning with non-blocking reduction

Parallel performance: 1 Mdof 2D Poisson



- In sequential extra computation makes p-CG (green curve) slower
- Quickly parallel p-CG outperforms regular parallel CG (blue curve)

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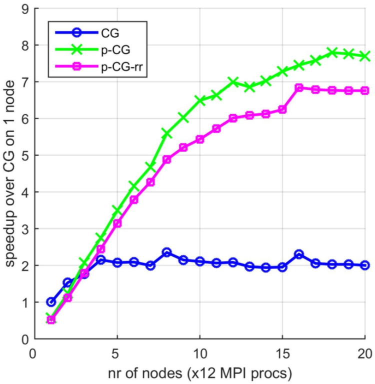

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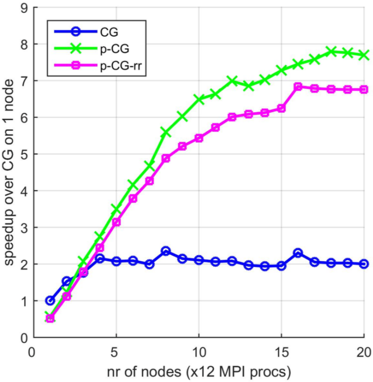
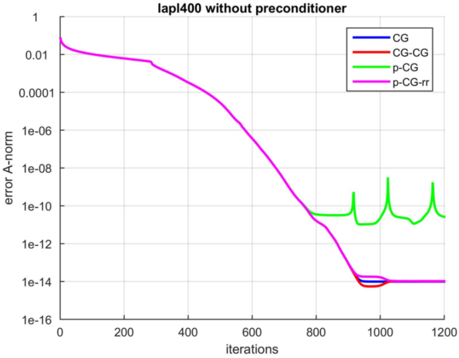


- In sequential extra computation makes p-CG (green curve) slower
- Quickly parallel p-CG outperforms regular parallel CG (blue curve)
- Teasing: purple curve

Why shall we mind ?



- Attainable accuracy of p-CG worse than classical CG
- Known/expected behaviour for three-term recurrence variants
  [M.H.Gutknecht, Z.Strakoš, SIMAX, 2000]

Possible remedy

- Develop a (tedious) rounding-error analysis based on known results (see e.g. N. Higham, SIAM book, 2002]) to compute the propagation of local rounding errors in pipelined CG

$$\mathrm{fl}(a \text{ op } b) = (a \text{ op } b)(1+\epsilon), \quad |\epsilon| \leq \psi$$

$$\begin{aligned} f_{i+1} &= (b - A\bar{x}_{i+1}) - \bar{r}_{i+1} \\ &= b - A(\bar{x}_i + \bar{\alpha}_i \bar{p}_i + \delta_i^x) - (\bar{r}_i - \bar{\alpha}_i \bar{s}_i + \delta_i^r) \\ &= f_i - \bar{\alpha}_i g_i - A\delta_i^x - \delta_i^r \end{aligned}$$

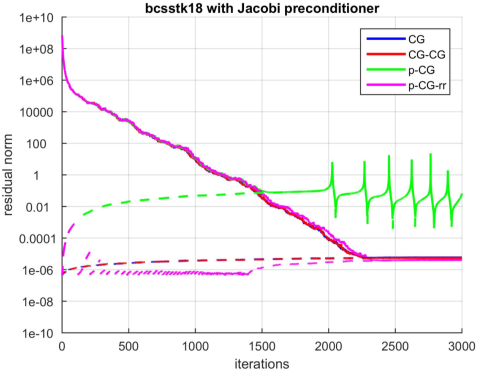
- Design a residual replacement strategy [H. van der Vorst, Q. Ye, SISC, 2000]

$$\|f_i\| \leq \sqrt{\psi}\|\bar{r}_i\| \quad \text{and} \quad \|f_{i+1}\| > \sqrt{\psi}\|\bar{r}_{i+1}\|.$$

Features of the new algorithm

At a negligible extra computational cost

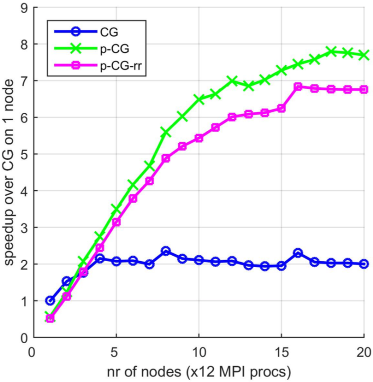
- Attainable accuracy is recovered



Features of the new algorithm

At a negligible extra computational cost

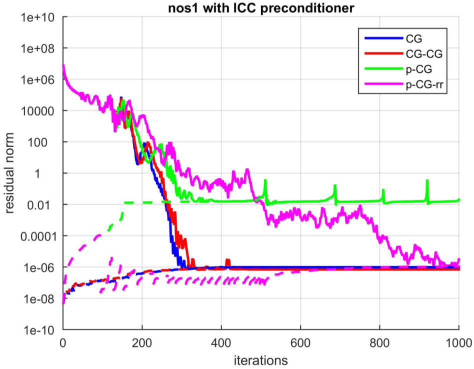
- Attainable accuracy is recovered
- Parrallel performance not “much affected” (purple vs green)



Not ended story

A few still open questions

- Analysis of the convergenve delay
- Relax some theoretical hypothesis that might not hold in practice



Outline

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Why soft errors occur?

What is soft error?

- Possible causes : voltage reduction, electricity fluctuations, cosmic particle effects, etc...
- Appears on: memories, registers, pipeline of the processor

Extreme scale platforms

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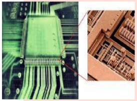
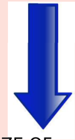
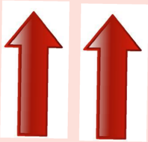




SIZE OF DEVICES

POSSIBLE SOFT ERROR RATES



OF COMPONENTS

AREA EFFECTED BY RADIATION

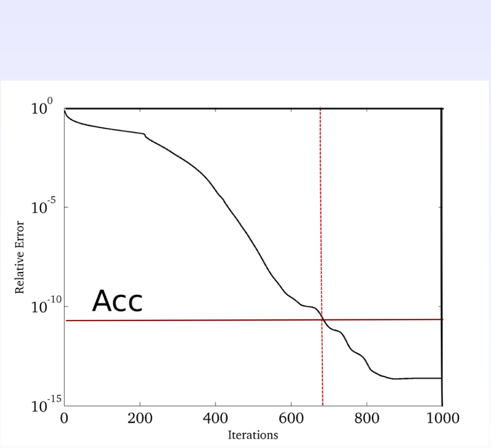
How soft errors occur?

```
for i = 0, ... do
    s := A p_i
    α := r_i^T u_i / s^T p_i
    x_{i+1} := x_i + α p_i
    r_{i+1} := r_i − α s
    u_{i+1} := M^{-1} r_{i+1}
    β := r_{i+1}^T u_{i+1} / r_i^T u_i
    p_{i+1} := u_{i+1} + β p_i
end for
```

We consider transient soft errors

- in the most computationally expensive kernels
- other "cheaper" kernels could be protected by redundancy

Protocol for sensitivity study



	Effect on the convergence
Converged	
Not Converged	

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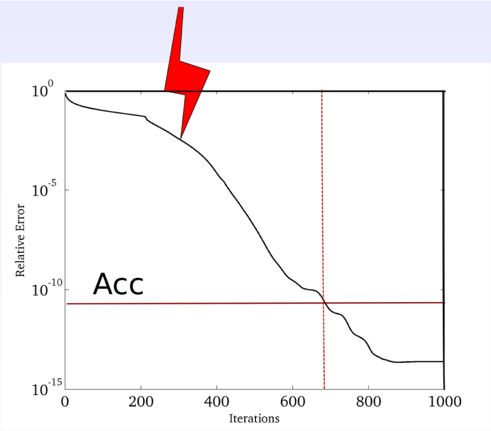

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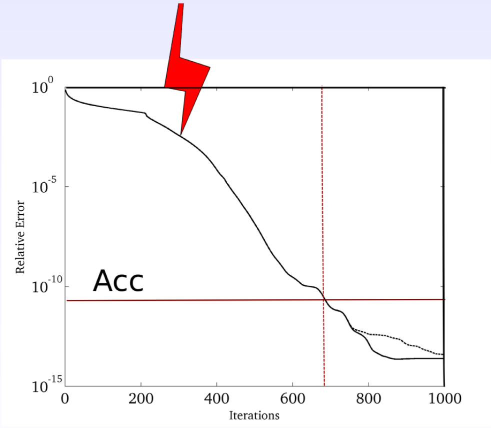

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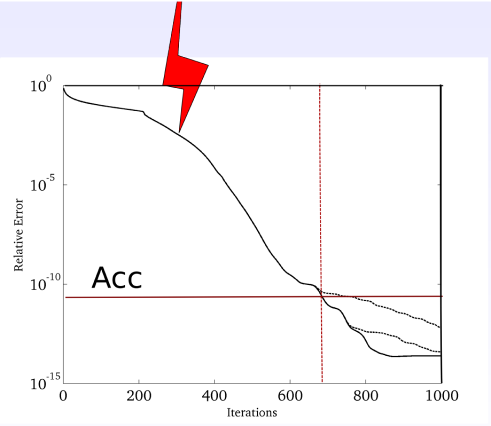

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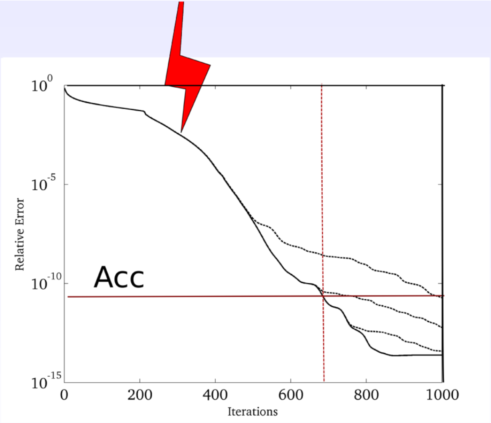

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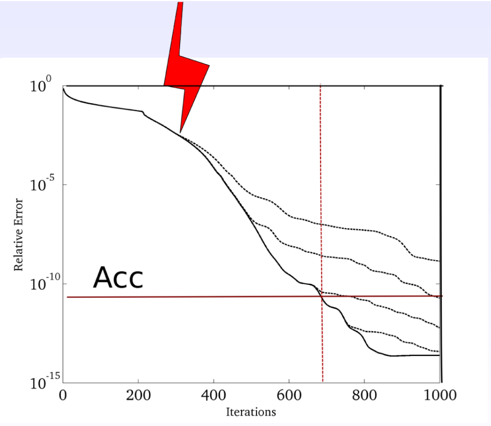

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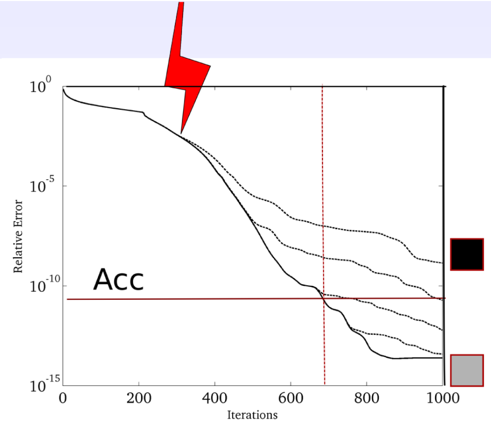

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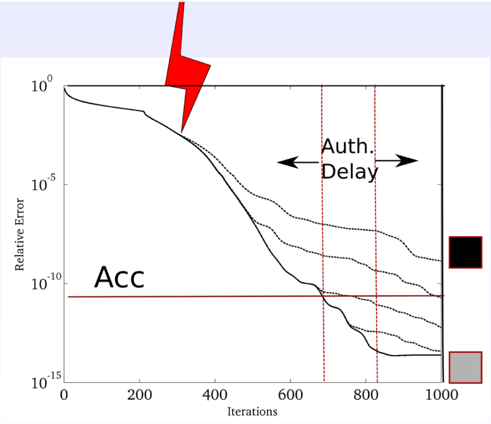

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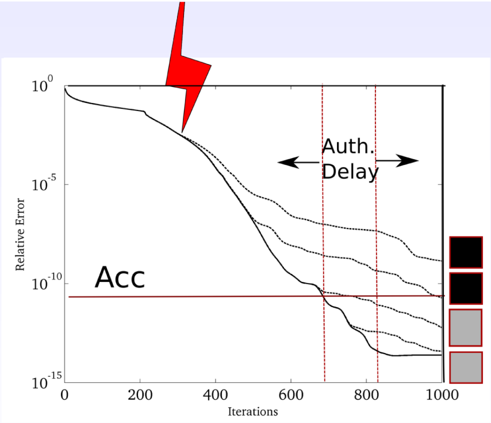

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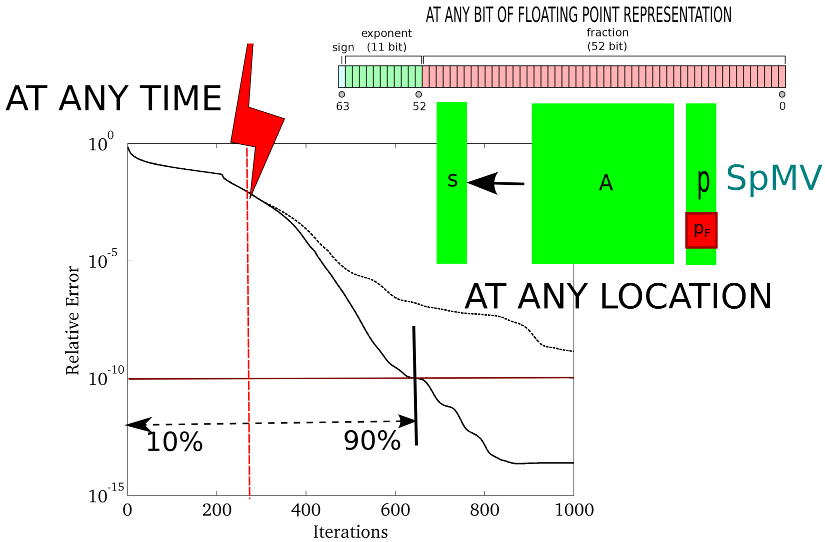
Fault injection methodology in 64-bit



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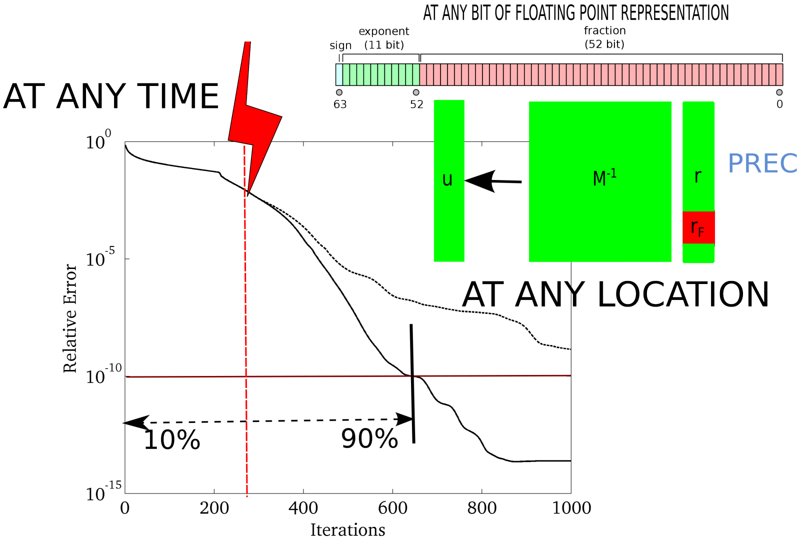
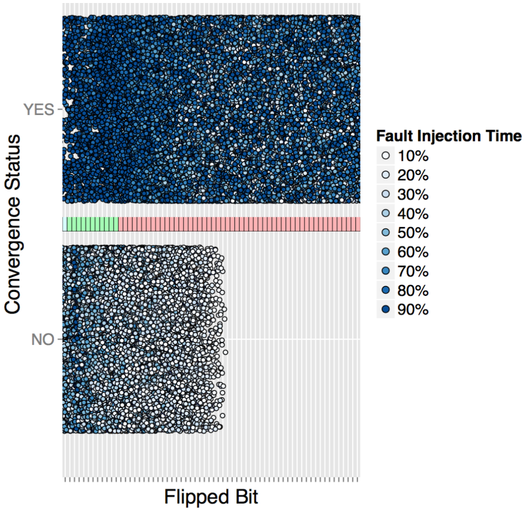
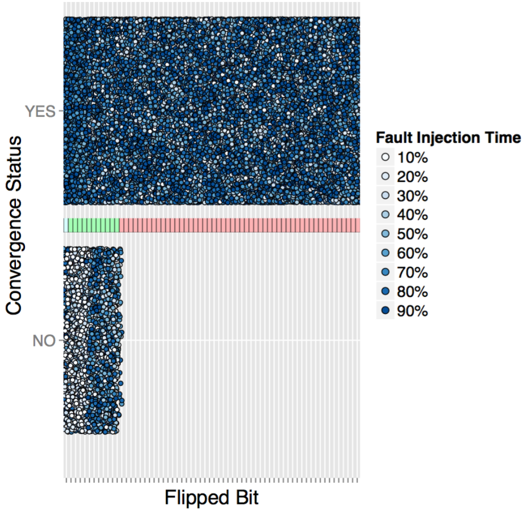


Sensitivity to soft-errors in mat-vec



- Many soft-errors are silent
- Mainly early bit-flip on high order bits are critical when computing $s_i = Ap_i$

Sensitivity to soft-errors in precond



- Even many more soft-errors are silent
- Mostly bit-flip in sign/exponent are critical when computing $u_{i+1} := M^{-1} r_{i+1}$

Main Observations

Sensitivity

- PCG algorithm is more sensitive for soft-errors on SpMV
- Soft-errors have different propagation patterns which influence the parameters of the algorithm in distinct ways

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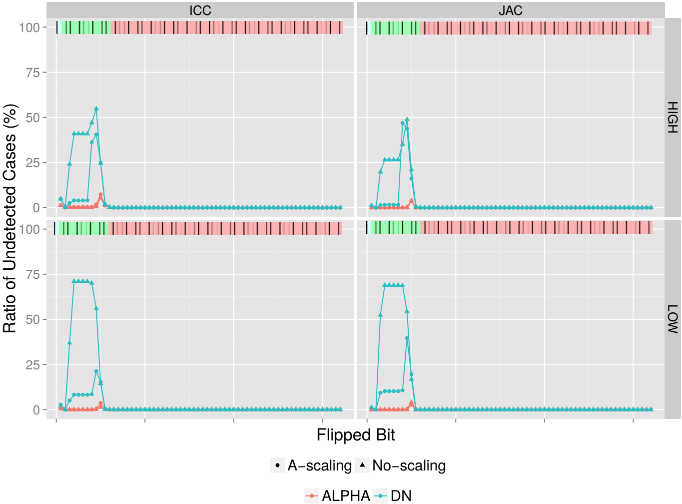
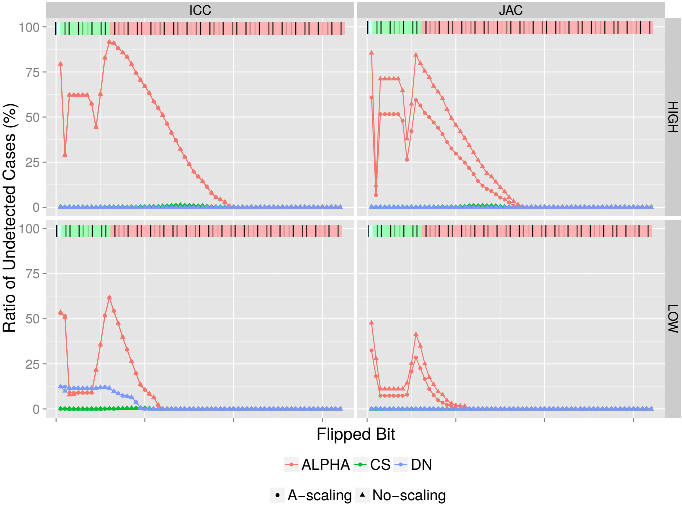
What about soft-error detection based on rounding error analysis [Vorst & Yee, SISC, 2000] a safe interval for α parameter $(\lambda_{\text{max}}^{-1} \leq \alpha_i \leq \lambda_{\text{min}}^{-1})$ [Hestenes & Steifel, :1 JNRBS, 1952] ?

Detection robustness v.s. Preco. bit-flip location



Detection robustness v.s. SpMV bit-flip location



Main Observations

Detection

- Deviation is a good criterion candidate for SpMV faults but not for preconditioner faults
- Control frequency for deviation should be investigated
- α criterion works well for preconditioner faults but not for SpMV
- An estimation of extremal eigenvalues of preconditioned matrix is needed for α criterion (often known for scalable preconditioners)

Acknowlegement for financial support:

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- G8 : ECS project

More information on
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Merci for your attention

Questions ?

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