QUANDELA

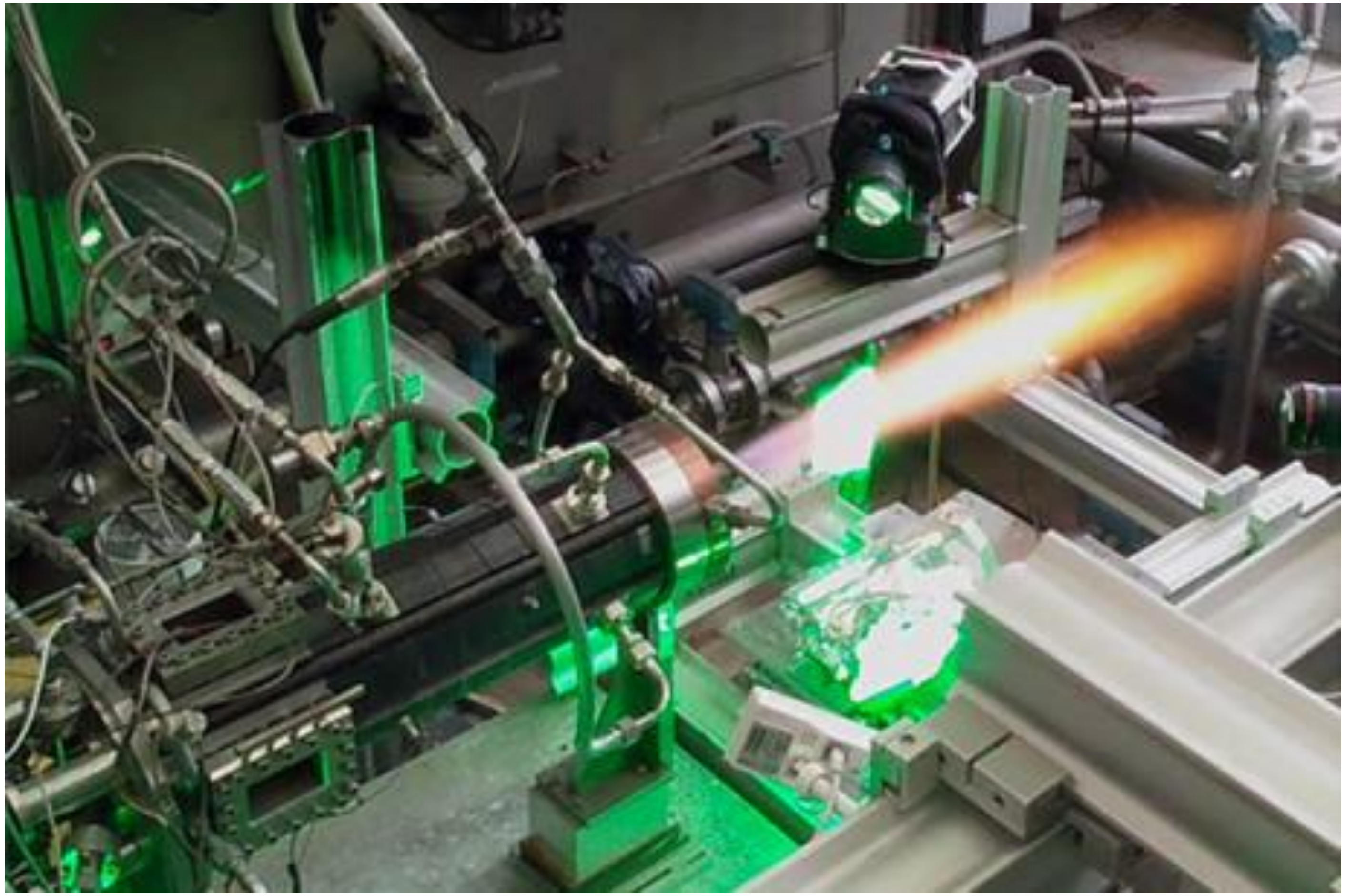
ONERA

THE FRENCH AEROSPACE LAB

AQCMA - Quantum advantage for aerospace engines design

Solving combustion differential equation on a photonic quantum neural network

Solving differential equations for fluid mechanics represent 95% of ONERA internal use of computing time



- Fluid mechanics problems are separated in two : combustion and aerodynamics.

- Combustion alone requires a lot of computing power, thus **electricity bill is consequential.**

- Combustion reaction are hard multi-physics problem. Better understanding of them would lead to **engines polluting less with a better efficiency.**

Ref : Challenges in combustion for Aerospace Propulsion, https://aerospacelab.onera.fr/al11

Problem statement : 1D-laminar flame equation with 2 reactants

Problem statement :

$$\partial_x \dot{m} = 0$$

$$\dot{m}\partial_x Y_1 - \frac{\lambda}{c_p} Le^{-1} \partial_x^2 Y_1 = -\tilde{A} T Y_1$$

$$\dot{m}\partial_x T - \frac{\lambda}{c_p} \partial_x^2 T = \tilde{\Delta}_h \tilde{A} T Y_1,$$

$$\dot{m} Y_1(-\infty) = \dot{m} Y_1(x_0) - \frac{\lambda}{c_p} Le^{-1} \partial_x Y_1(x_0)$$

$$\dot{m} T(-\infty) = \dot{m} T(x_0) - \frac{\lambda}{c_p} \partial_x T(x_0)$$

$$\partial_x Y_1(x_N) = 0$$

$$\partial_x T(x_N) = 0,$$

Classical solution :

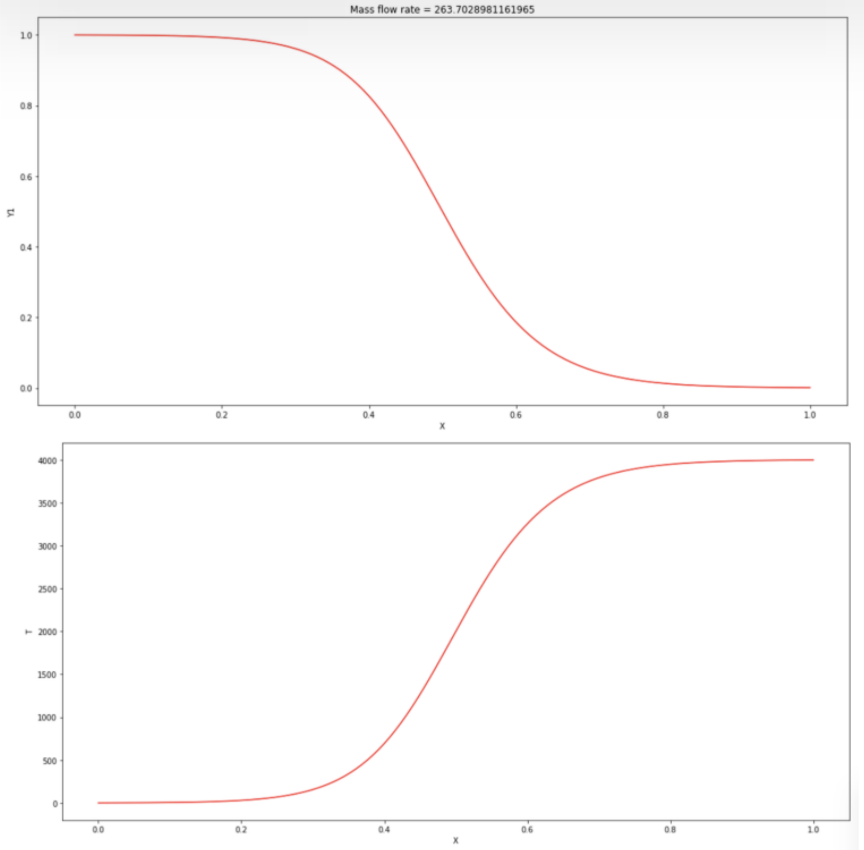


FIG. 1: Solution obtained after running the code from the Appendix. This plot is reached after 8 steps in the Newton-Raphson method, for a grid size of 256.

QML architecture : a universal Fourier series approximator

Circuit used :

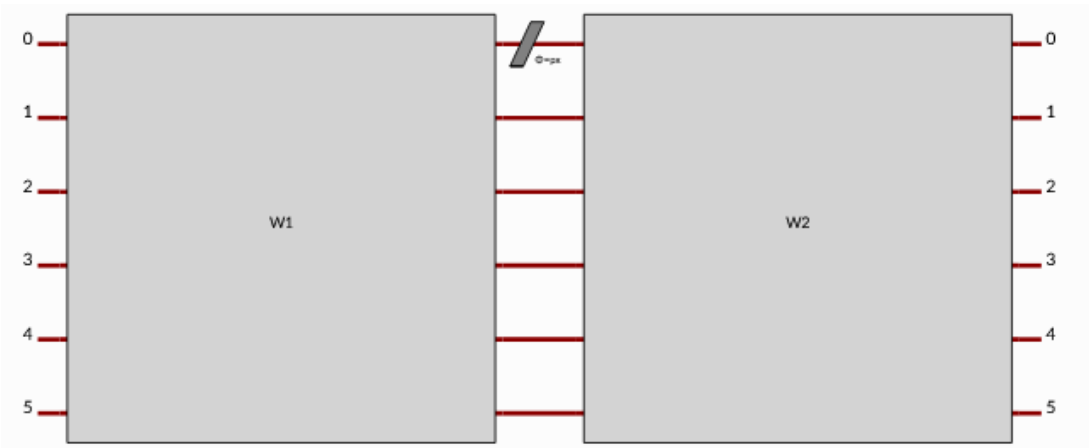


Fig. Photonic QML ansatz being a universal fourier approximator.

Vector of number of photons in each mode

$$n = (n_0, n_1, \dots, n_5)$$

Parametrized beam-splitters meshes :

$$W_1(\theta_1), W_2(\theta_2) \text{ and } \Theta = (\theta_1, \theta_2)$$

x is the input data as well as the parameter of the phase shifter.

Output :

Fourier decomposition:

$$f_n(x, \Theta, \lambda) = \sum_{\omega \in \Omega_n}^{N} c_\omega(\Theta, \lambda) e^{i\omega . x}$$

With $\Omega_n = [\![-n; n]\!]$ the frequency spectrum one can reach with n photons and $c_\omega(\Theta, \lambda)$ the Fourier coefficients.

Ref : Fock State-enhanced Expressivity of Quantum Machine Learning Models Beng Yee Gan, Daniel Leykam, and Dimitris G. Angelakis

QML architecture : a universal Fourier series approximator to solve differential equations

Loss function for differential equation, $\frac{df}{dx} + \lambda f(x)(\kappa + \tan(\lambda x)) = 0, f(0) = f_0$:

$$(\Theta_{opt}, \lambda_{opt}) = \underset{\Theta,\Lambda}{\text{argmin}} \frac{1}{M} \sum_{i=1}^{M} L(\frac{df_n(x_i, \Theta, \lambda)}{dx} + \lambda f_n(x_i, \Theta, \lambda)(\kappa + \tan(\lambda x)), 0) + \eta L(f_n(0, \Theta, \lambda), f_0)$$

With $L(a, b) = (a - b)^2$ for $(a, b) \epsilon \mathbb{R}^2$, and a function f_n discretized on M points.

Outcome :

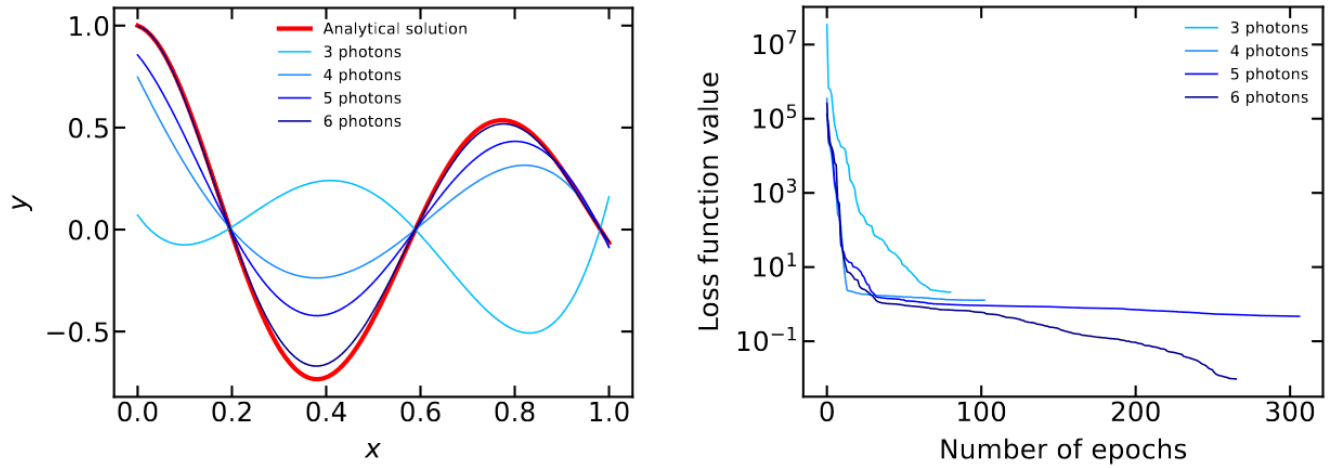


Fig. PDE resolution with a photonic QML ansatz and varying number of photons.
On the left the outputted function. On the right the loss function vs number of epochs.

Resolution of heat equations using photonic QML

Reply sigma solved a 2D heat equation:

$$\frac{\partial u}{\partial t} = k\left[\frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial^2 y}\right]$$

with constant boundary and initial conditions

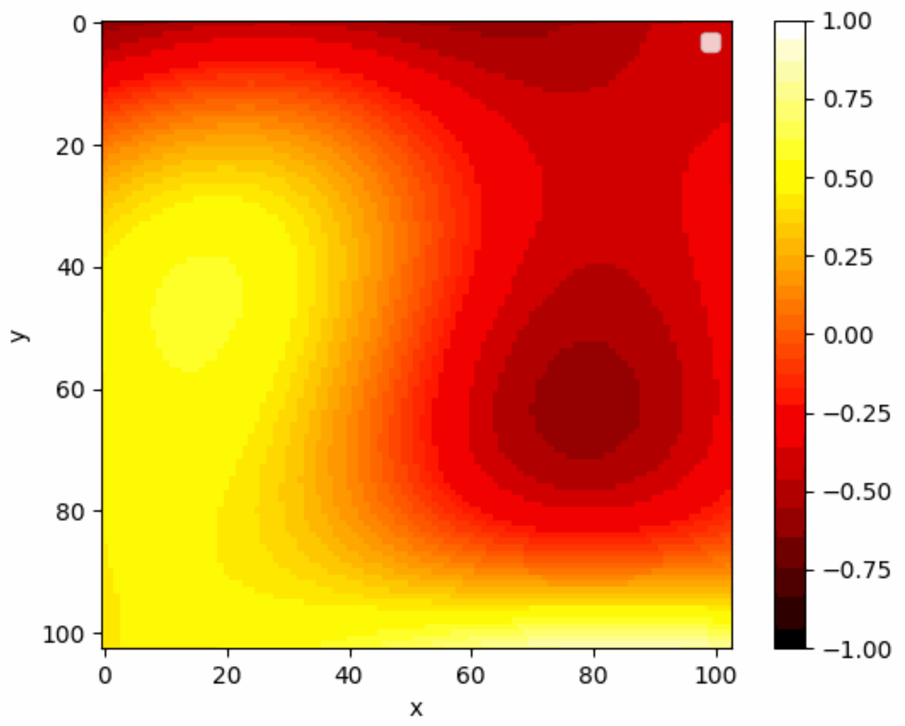


Fig. Resolution of a 2D-heat equation with photonic QML ansatz credits to Brian Ventura and Hugo Jaunin from Reply Sigma

Next steps : enhancement of photonic neural network and runs on QPU

i) Enhance photonic neural networks

- Add photon number resolution
- Add data-reuploading trick

To solve the couple system of combustion reaction

ii) Runs on QPU

$$\partial_x \dot{m} = 0$$
$$\dot{m}\partial_x Y_1 - \frac{\lambda}{c_p} Le^{-1} \partial_x^2 Y_1 = -\tilde{A} T Y_1$$
$$\dot{m}\partial_x T - \frac{\lambda}{c_p} \partial_x^2 T = \tilde{\Delta}_h \tilde{A} T Y_1,$$

Fig. Coupled system of differential equations to be solved

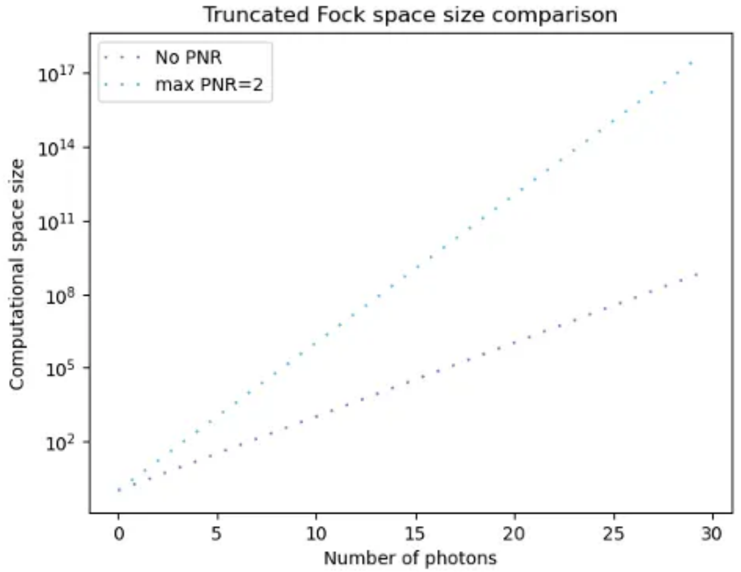


Fig. Computational space size with and without pseudo-PNR detectors

Ref : More about PPNRs on this medium post : https://medium.com/quandela/budgeting-with-qubits-make-the-most-of-your-photons-2519f06b5810 . More about their usefullness in QML here : https://medium.com/quandela/photonics-the-fast-lane-towards-useful-quantum-machine-learning-f3112f7221fc

Recruitment and knowledge transfer

2 internships of six months recruited by ONERA for AQCMA

2 internships of six months recruited by Quandela for AQCMA

ONERA staff learned how to use Perceval

1 full-time employee to be hired on solving differential equations using Quantum Computers

1 application engineer working part-time on it and an expert on solving differential equations using Quantum Computers just joined the applications team

Work breakdown structure