

Twins: From concept to success stories

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ARTS
ET MÉTIERS
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I - CONCEPT



$$\mathcal{L}(u(x, t; \mu)) = f(x, t; \eta)$$

$$\text{B.C. } (x, t; \xi)$$



Calibration
Data
 μ



Process
 η, ξ



$$u(x, t) \approx \sum_{i=1}^N U_i(t) N_i(x) \xrightarrow{\text{red arrow}} \mathbf{KU} = \mathbf{F}$$

Well Experienced Engineering

Nominal calibrated model
Nominal loading



DESIGN
(Nominal)
in service

Virtual Twin

Real-Time Decision Making



Nominal calibrated model
Nominal loading



DESIGN
(Nominal)
in service

DATA

- Inputs
- Outputs

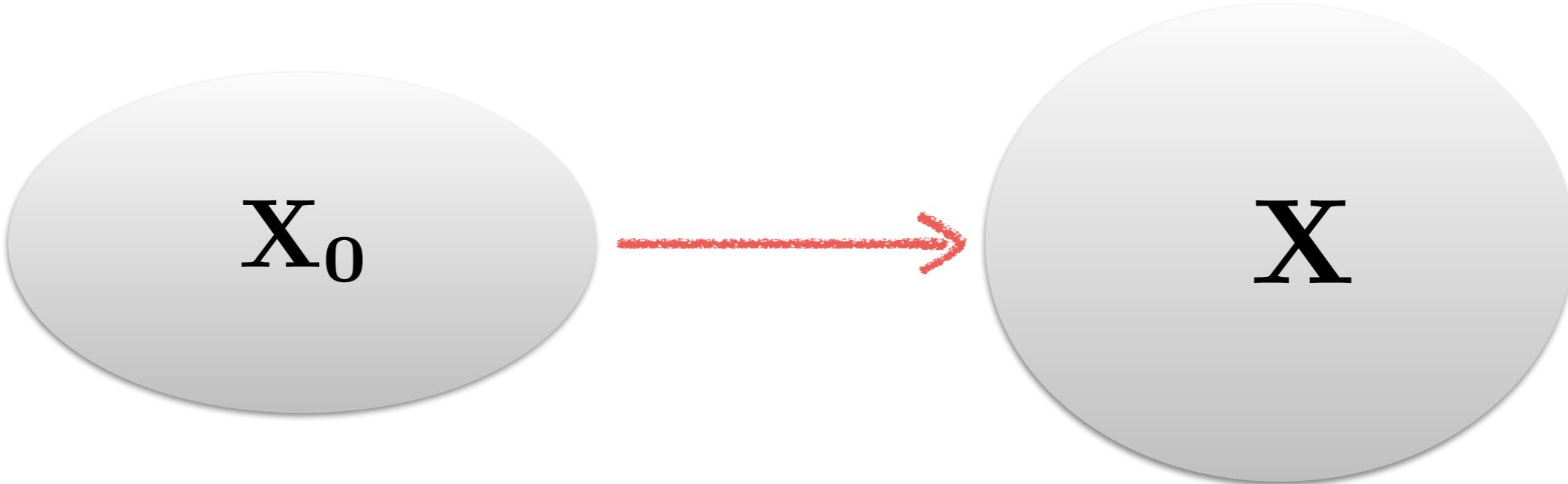
Learning

Deep learning
Tensor learning,
Dictionary learning,
Manifold learning,
Linear and nonlinear regression
Sparse identification
DMD & Koopman operator,
Data-driven operator



$O(\mathcal{I})$
ONLINE
Digital Twin

Data-Driven Model Adaptation

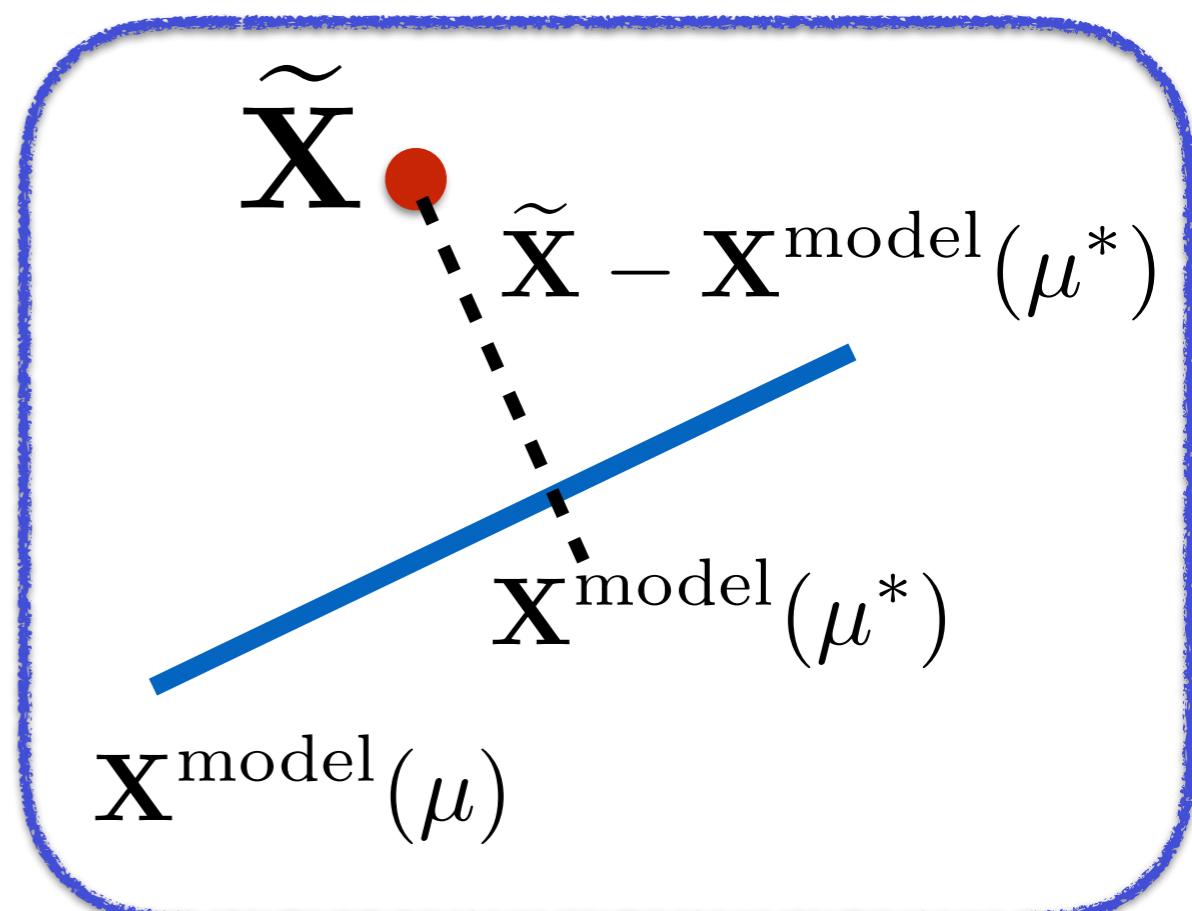


Hybrid Twin

Parametric model $\mathbf{X}^{\text{model}}(\mu)$

Ignorance $\tilde{\mathbf{X}} - \mathbf{X}^{\text{model}}(\mu^*)$

Data-Based
Divergence Model



HYBRID-TWIN

Parametric
deterministic
contribution
PGD

Data-Driven
correction

Control

Noise

$\left\{ \begin{array}{l} \dot{\mathbf{X}} = \mathbf{A}(\mathbf{X}, t, \mu) + \mathbf{B}(\mathbf{X}, t) + \mathbf{C}(\mathbf{X}) + \mathbf{R} \\ \mathbf{Y} = \mathbf{D}(\mathbf{X}) + \mathbf{R}' \\ \mathbf{Z} = \mathbf{G}(\mathbf{X}) + \mathbf{R}'' \end{array} \right. \quad \text{Measures involving noise}$

Filters:
Kalman,
Bayesian,
...

```
graph LR; PGD[PGD] --- A["dot X = A(X, t, mu) + B(X, t) + C(X) + R"]; DDC[Data-Driven correction] --- D["Y = D(X) + R'"]; Control[Control] --- G["Z = G(X) + R'"]; Noise[Noise] --- R[""]; subgraph Measures [Measures involving noise]; DDC --- D; Control --- G; Noise --- R; end; PGD --- A;
```

If the physics-based model is accurate enough, the deviation remains small, and its associated data-driven model results almost linear (or moderately nonlinear). Thus, little data suffices to infer it (the model correction).

Data is expensive to collect and to assimilate !

What is needed?

- Real-time simulation
- Real-time calibration
- Real-time data-assimilation
- Real-time data-completion
- Real-time data-analytics
- Real-time data-driven modeling
- Real-time decision making

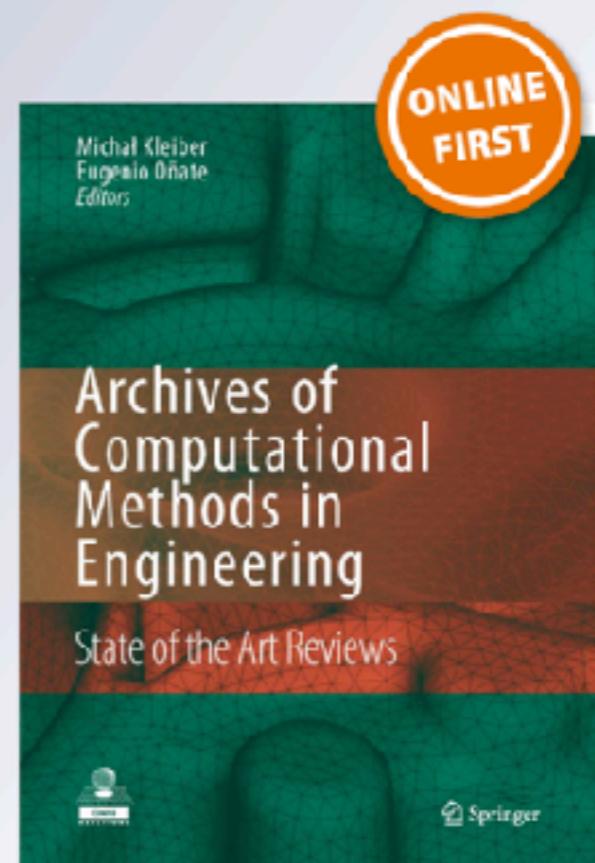
Virtual, Digital and Hybrid Twins: A New Paradigm in Data-Based Engineering and Engineered Data

**Francisco Chinesta, Elias Cueto,
Emmanuelle Abisset-Chavanne, Jean
Louis Duval & Fouad El Khaldi**

Archives of Computational Methods
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State of the Art Reviews

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 Springer

PGD based real-time simulation

Parametric solutions



Intrusive constructor, or
Non-intrusive: SSL, sPGD, ...

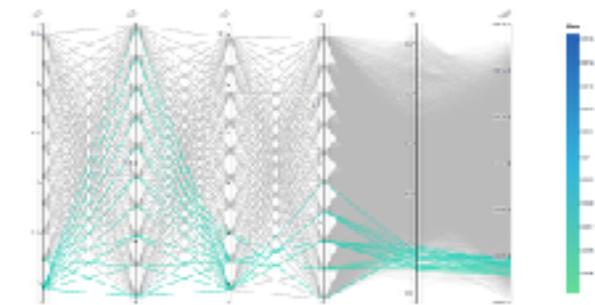
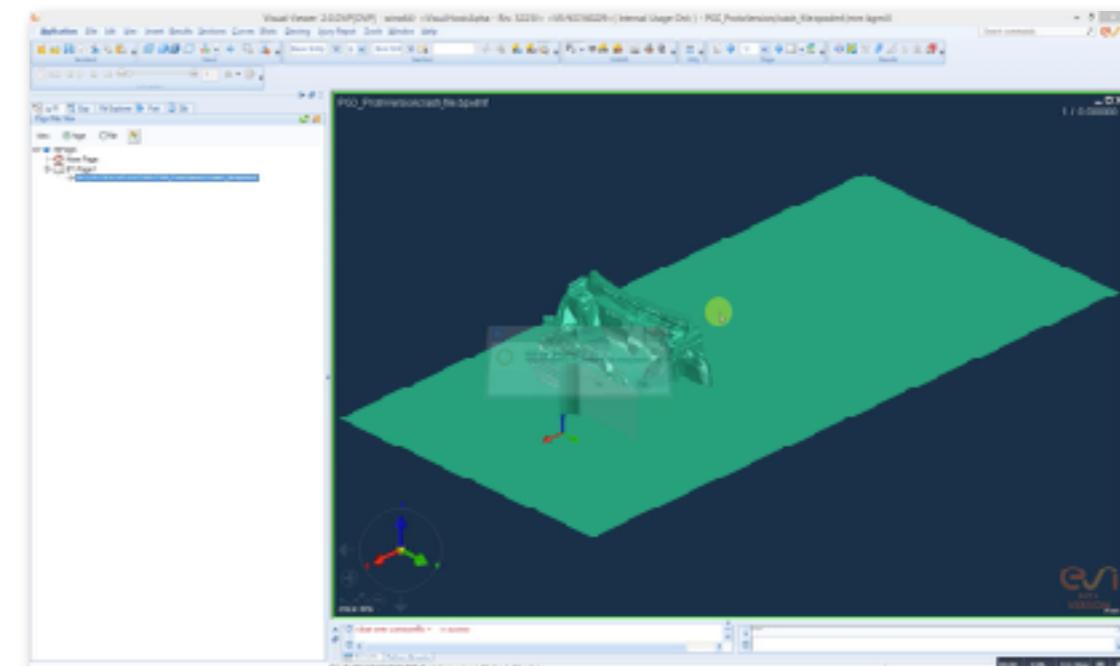
$$u(x, t, p) \approx \sum_{i=1}^M T_i(t) X_i(x) P_i(p)$$

ONLINE



Almost

- Real-time simulation
- Real-time optimization
- Real-time inverse analysis
- Real-time uncertainty propagation
- Real-time control

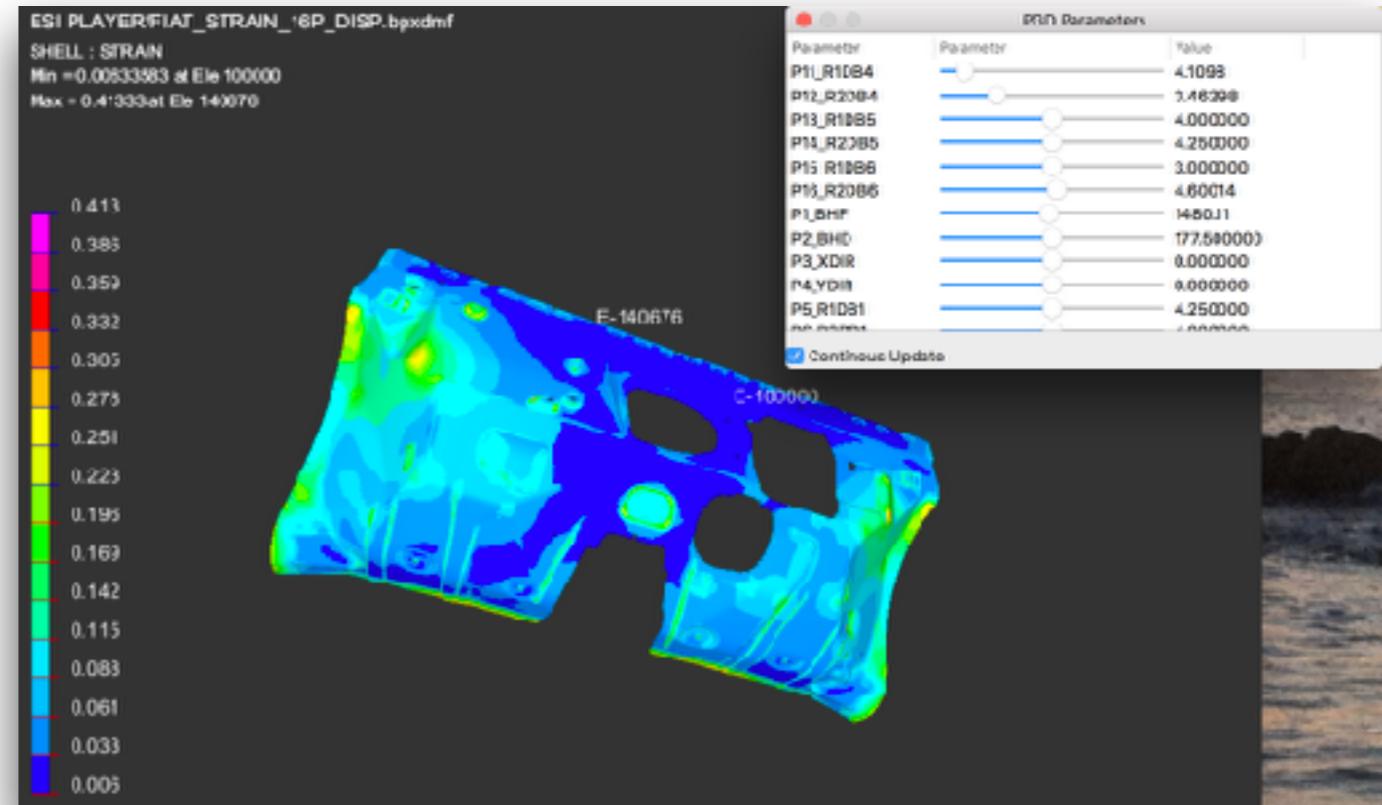


PGD based real-time simulation

PGD (variables separation) + Sparse Sensing + Kriging



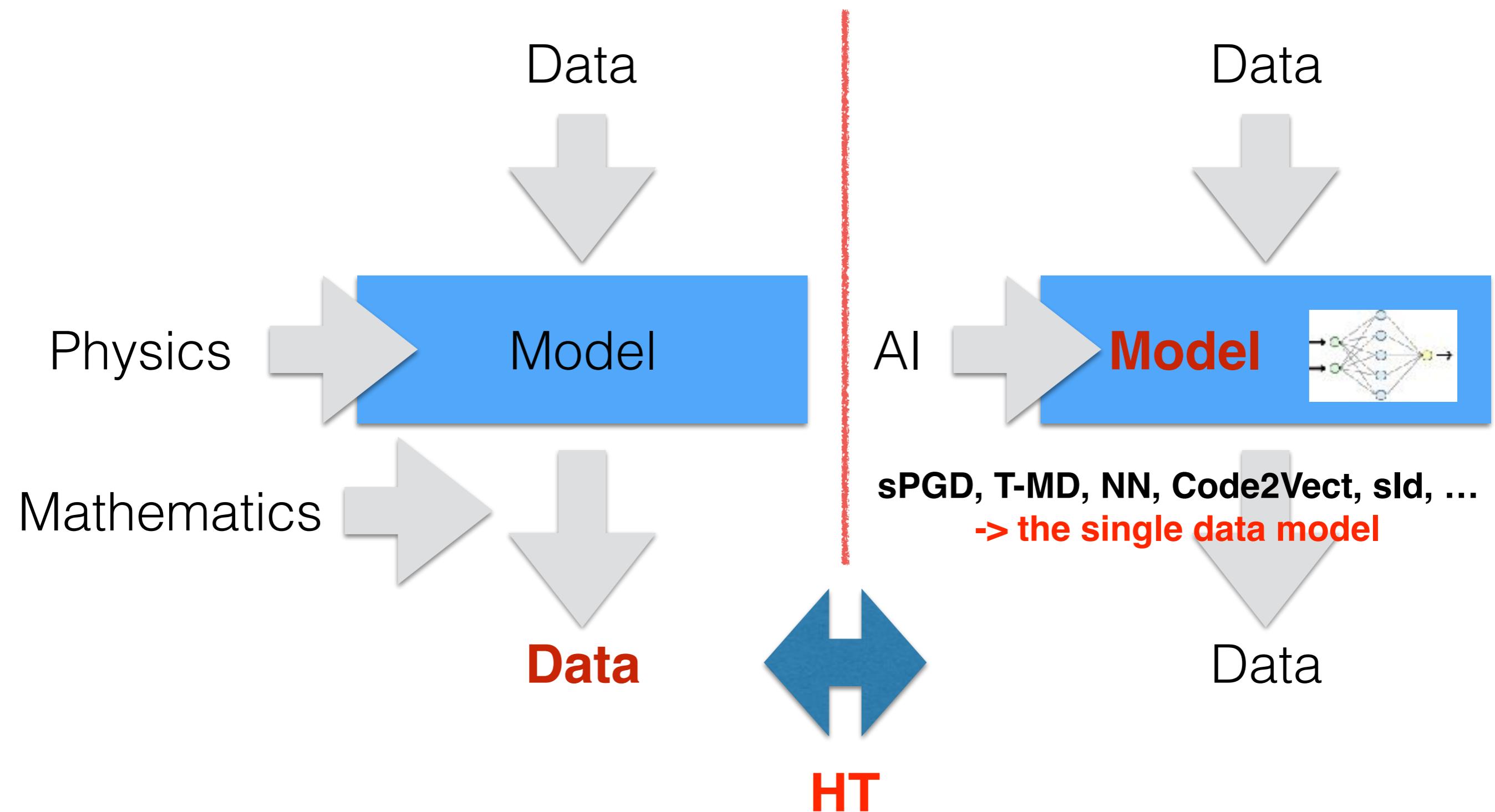
15 parameters / 14 runs



PROS:

- Non-intrusive
- P parameters P runs
- Is being validated

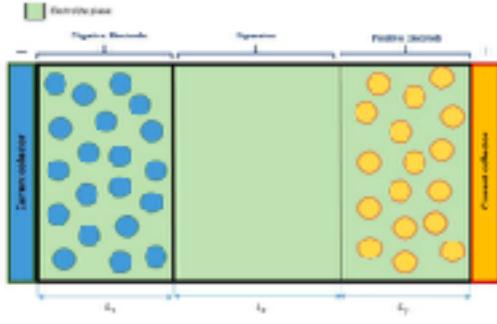
Data-Based Physics & Data-Driven Modelling



II - INDUSTRIAL SUCCESS

Case-study I: Batteries

Micro



Newman's P2D model

$$\frac{\partial \psi}{\partial t} = \frac{D_1}{x^2} \frac{\partial}{\partial x} \left(r^2 \frac{\partial \psi}{\partial r} \right); \quad \frac{\partial \psi_1}{\partial r} \Big|_{r=0} = 0; \quad D_2 \frac{\partial \psi_2}{\partial r} \Big|_{r=R_2} = -\frac{\sigma_2}{\alpha_2} I$$

$$\frac{\partial}{\partial x} \left(k^2 f^2 \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial z} \left(k^2 f^2 \frac{\partial \phi}{\partial z} \ln(c_e) \right) + I^{(2)} = 0; \quad \frac{\partial \phi_2}{\partial x} \Big|_{x=0} = 0$$

$$\frac{\partial}{\partial x} \left(\sigma^2 f^2 \frac{\partial \phi}{\partial x} \right) + I^{(2)} = 0;$$

$$-\sigma^2 f^2 \frac{\partial \phi}{\partial x} \Big|_{x=0} - \sigma^2 f^2 \frac{\partial \phi}{\partial x} \Big|_{x=L} - \frac{I}{A} \cdot \frac{\partial \phi_2}{\partial x} \Big|_{x=0} - \frac{\partial \phi_2}{\partial x} \Big|_{x=L} = 0$$

$$\frac{\partial (\phi_2 \psi_2)}{\partial r} = \frac{\partial}{\partial x} \left(D_2 \frac{\partial \phi_2}{\partial x} \right) + \frac{I^{(2)}}{c_e}; \quad \frac{\partial \phi_2}{\partial x} \Big|_{x=0} = \frac{\partial \phi_2}{\partial x} \Big|_{x=L}$$

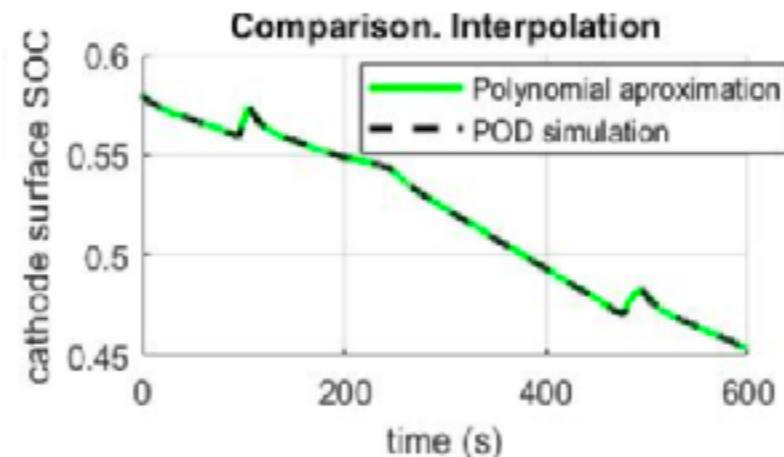
POD & PGD

Meso

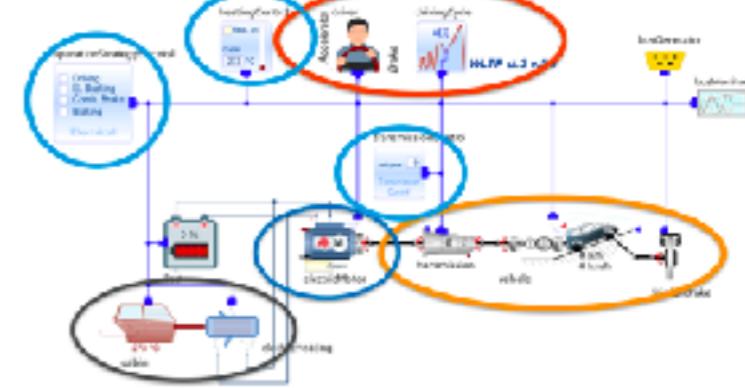
$$SOC(t) = f(SOC(t = 0), Load, Env)$$

$$Voltage(t) = g(SOC(t = 0), Load, Env)$$

AI: sPGD & T-MD

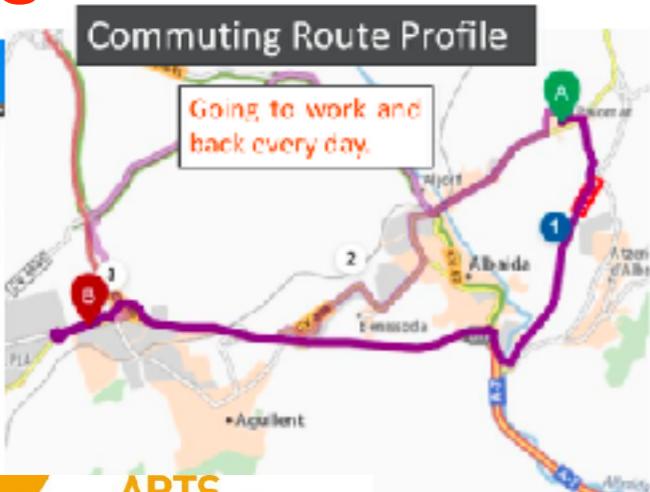


Macro

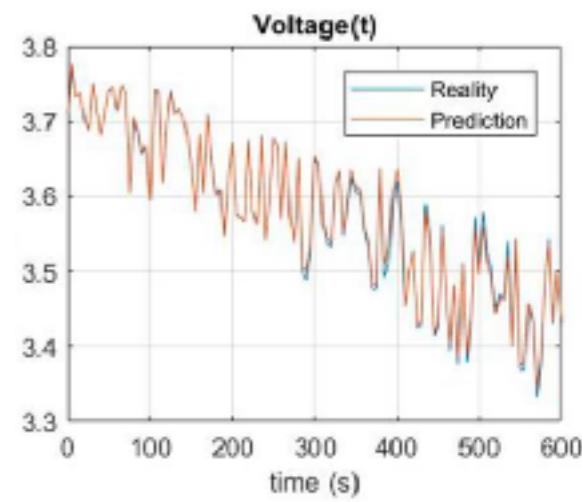
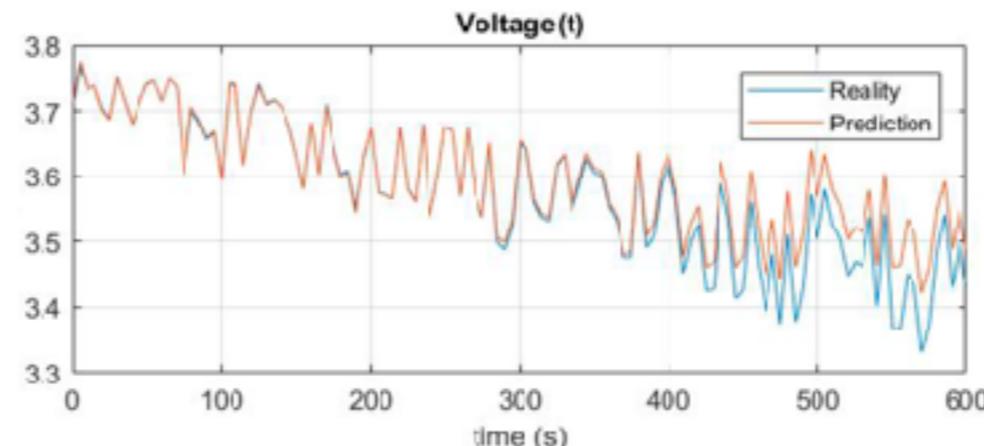


System modelling

Planning



HT

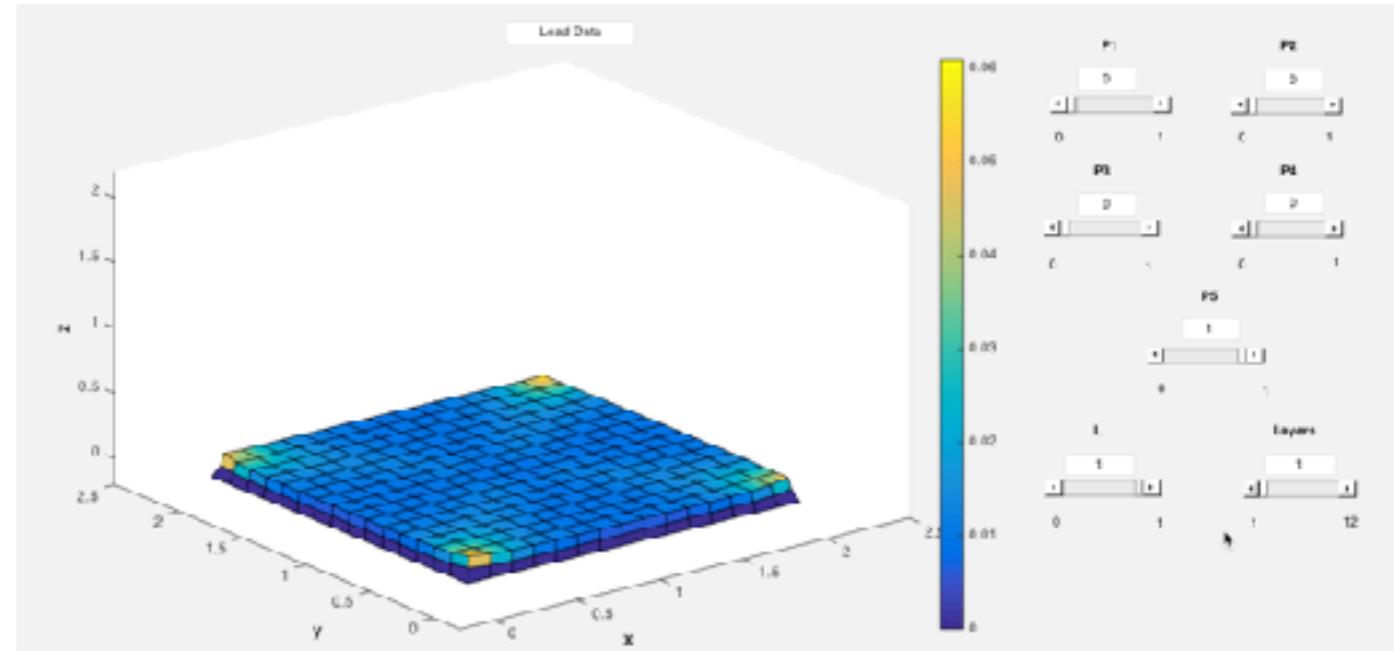


Data-Driven Correction

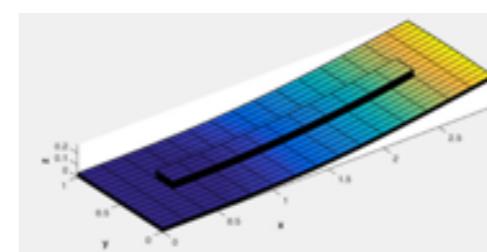
Case-study II: AM



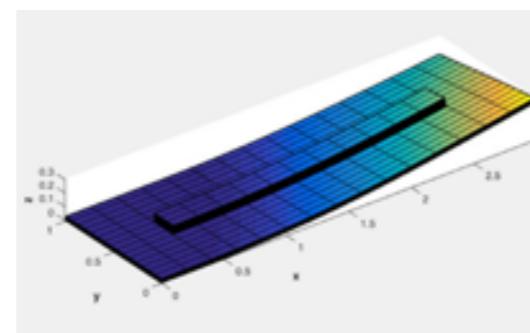
$$\Delta\phi = s$$
$$\phi|_{\partial\Omega} = f(\Phi_1, \Phi_2, \Phi_3, \Phi_4)$$
$$\tilde{\sigma}_0(\mathbf{P}) = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



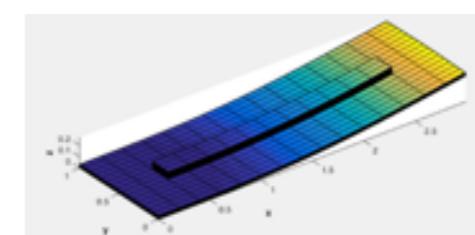
Hybrid Twin
Target



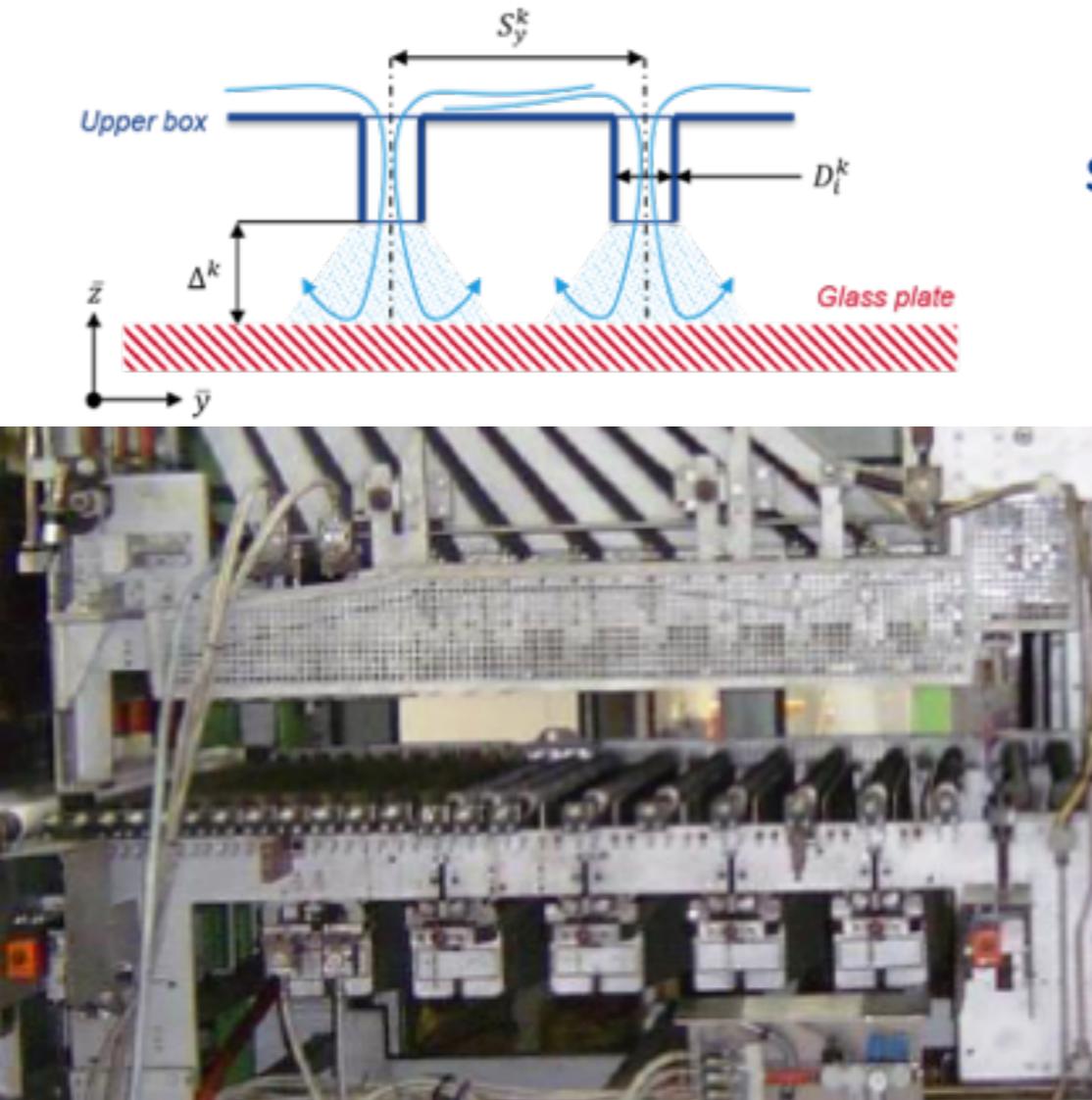
Anticipated
based on
data



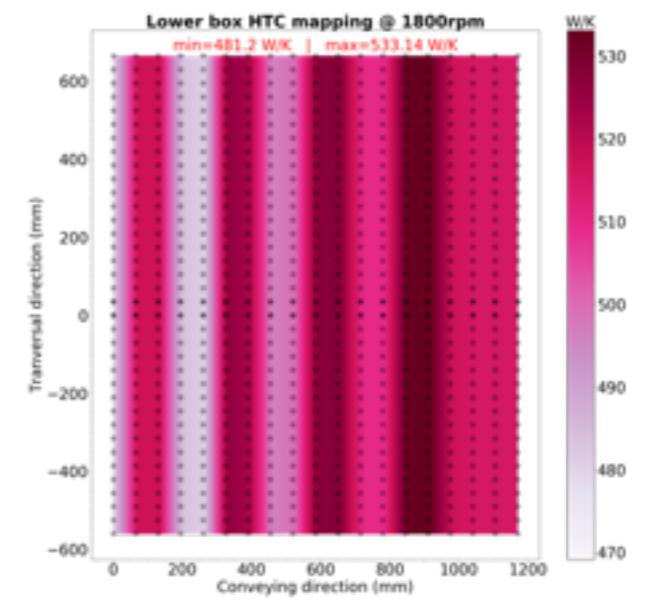
Correction
Result



Case-study III: Tempering



$$\Delta h_i^k = f(Re, Pr, D, \Delta, f)$$



HYBRYD TWIN =
PHYSICS BASED
(REAL-TIME) MODEL
(appl. math.) + DATA-
DRIVEN LEARNED IN-
THE-FLY MODEL (AI)