Function Maximization with Dynamic Quantum Search

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Table of Contents

The problem

$$\max_{x \in \{0/1\}^n} f(x)$$

$n = 4$ Knapsack example:

- Item 1 (7 kg, \$40)
- Item 2 (4 kg, \$100)
- Item 3 (2 kg, \$50)
- Item 4 (3 kg, \$30)

Knapsack 0/1 example

Table: Candidate solutions and values for Knapsack problem data.

Candidate	Fitness / 10	Weight	Validity
0 0 0 0	0	0	valid
0 0 0 1	3	3	valid
0 0 1 0	5	2	valid
0 0 1 1	8	5	valid
0 1 0 0	10	4	valid
0 1 0 1	13	7	valid
0 1 1 0	15	6	valid
0 1 1 1	**18**	**9**	**valid**
1 0 0 0	4	7	valid
1 0 0 1	7	10	valid
1 0 1 0	9	9	valid
1 0 1 1	12	12	invalid
1 1 0 0	14	11	invalid
1 1 0 1	17	14	invalid
1 1 1 0	19	13	invalid
1 1 1 1	22	16	invalid

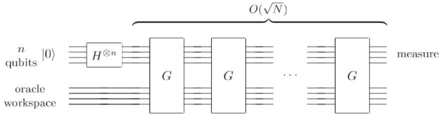
Interesting quantum properties

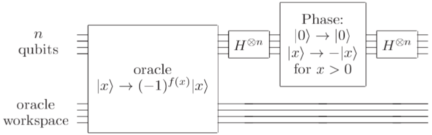
- Superposition : $\sum_{x=0}^{2^n-1} a_x|x\rangle$
- Parallel application : $\sum_{x=0}^{2^n-1} a_x|x\rangle|0\rangle \rightarrow \sum_{x=0}^{2^n-1} a_x|x\rangle|f(x)\rangle$
- Quadratic speedup with Quantum Search (Grover's algorithm - 1996)
- Adaptation for finding the maximum (Ahuja, Kapoor - 1999)

Quantum Search

Quantum search in literature with a list P of $N = 2^n$ elements:

- Start with : $\frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$
- Specify an oracle : $f(x) = 1$ if x is the seeked element, 0 otherwise
- Apply Grover iterations $O(\sqrt{N})$ times
- Measure with a high probability to get the seeked x.





Quantum Search of the Maximum

- For finding the maximum, same process but input an index k for the maximum :

$$\frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle|k\rangle$$

- Specify an oracle : $f(x) = 1$ if $P[x] > P[k]$, 0 otherwise
- Apply iterations and measure a new k
- Complexity $\approx 13.6\sqrt{N}$

Table of Contents

Initial requirements

Specify registers :

- q: an n-qubit register for storing candidate solutions. $n = 4$ in this example.
- f: a p-qubit register for storing the value of the fitness calculation in two's complement representation, where p is sufficient to store the sum of all possible values. $p = 6$ is this example.
- w: a register for storing the total mass of a candidate backpack. $\dim(w) = 5$ using unsigned integer representation in this example.
- g: a register for storing intermediate computational states and numerical constants. $\dim(g) = 6$ for this example.
- v: a 1-qubit register to mark a candidate fitness as valid or invalid.
- r: a 1-qubit register used by the oracle.

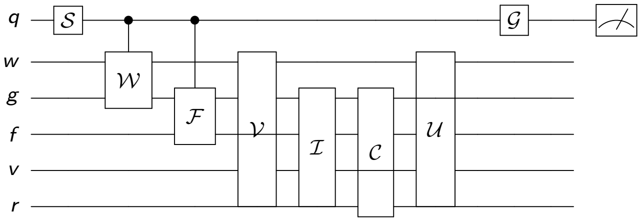
Arithmetic quantum circuits for the oracle :

- Let a, b be integers in m-bitstring format $\{a_i, b_i\}_{i=0}^{m-1}$, $a = \sum_{i=0}^{m-1} a_i 2^i$
- Quantum ripple adder : $|a, b\rangle \rightarrow |a, a+b\rangle$
- Controlled version : $|c, a, b\rangle \rightarrow |c = 0, a, b\rangle, |c = 1, a, a+b\rangle$
- Comparator : $|a, b, 0\rangle \rightarrow |a, b, 1\rangle$ if $a < b$

Building the oracle : Program

1. $\frac{1}{\sqrt{16}} \sum_{i=0}^{15} |i\rangle_q |0\rangle_f |0\rangle_w |0\rangle_g |0\rangle_v |-\rangle_r$
2. Control-add w_1 : $(\sum_{i,i_0=0} |i\rangle_q |0\rangle_w + \sum_{i,i_0=1} |i\rangle |0 + w_1\rangle_w) |w_1\rangle_g$
3. Control-add w_2, w_3, w_4 : $\sum_{i=0}^{15} |i\rangle_q |0\rangle_f |m(i)\rangle_w$
4. Control-add f_1, f_2, f_3, f_4 : $\sum_{i=0}^{15} |i\rangle_q |f(i)\rangle_f |m(i)\rangle_w$
5. Define validity with comparator :
$$\sum_{i,m(i)\leq 10} |i\rangle_q |m(i)\rangle_w |10\rangle_g |0\rangle_v + \sum_{i,m(i)>10} |i\rangle_q |m(i)\rangle_w |10\rangle_g |1\rangle_v$$
6. Inverse fitness for invalid candidates (by complementing and adding 1):
$$\sum_{i,m(i)\leq 10} |i\rangle_q |f(i)\rangle_f |m(i)\rangle_w |0\rangle_g |0\rangle_v + \sum_{i,m(i)>10} |-f(i)\rangle_f |i\rangle_q |m(i)\rangle_w |0\rangle_g |1\rangle_v$$
7. Mark better candidates using a comparator :
$$\sum_{i,f(i)\leq k} |i\rangle_q |f(i)\rangle_f |k\rangle_g |-\rangle_r + \sum_{i,f(i)>k} -|i\rangle_q |f(i)\rangle_f |k\rangle_g |-\rangle_r$$
8. Uncompute :
$$\sum_{i,f(i)\leq k} |i\rangle_q |0\rangle_f |0\rangle_w |0\rangle_g |0\rangle_v |-\rangle_r + \sum_{i,f(i)>k} -|i\rangle_q |0\rangle_f |0\rangle_w |0\rangle_g |0\rangle_v |-\rangle_r$$

Circuit and Strategy



Quantum search for the case M is unknown (Boyer et al., 1999)

1. Initialize $m = 1$ and set a constant $\lambda = 6/5$.
2. Choose a random integer j that is less than m.
3. Apply j Grover routines over the superposition of candidates
4. Measure to get an outcome.
5. Update current max if better.
6. Else set m to $\min(\lambda m, \sqrt{N})$ and repeat from step 2.

Table of Contents

Simulation and Discussion

- Could get solution in 2-3 Grover iterations
- Need uncomputation and reversibility
- Alternative to or combine with standard classical techniques

Publication

Moussa C., Calandra H., Humble T.S. (2019) Function Maximization with Dynamic Quantum Search. In: Feld S., Linnhoff-Popien C. (eds) Quantum Technology and Optimization Problems. QTOP 2019. Lecture Notes in Computer Science, vol 11413. Springer, Cham

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Thank you!